A Synopsis On

" Some Contributions To Solve Integro-Differential Equations"

Submitted to

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For the Degree of

Doctor of Philosophy (Ph. D.)

In

Mathematics

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March-2022

Title: “Some Contributions To Solve Integro-Differential Equations”

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1. Introduction:

The subject of integral equations is one of the most useful mathematical tools in both pure and applied Mathematics. It has tremendous applications in numerous physical issues [1, 2, 3]. In recent years, there has been an increasing interest in the integral and integro-differential equations. The integro-differential equations play an important role in many branches of linear and non-linear functional analysis and their applications in the theory of engineering, physics, mechanics, chemistry, biology, economics and electrostatics [4, 5, 6].

Integro-differential equations appear in many scientific applications, especially when we convert initial value problems or boundary value problems to integral equations. The integro-differential equations contain both integral and differential operators.The derivatives of unknown functions may appear to any order.

$$u^{(n)}(x) = f(x) + \int_{g(x)}^{h(x)} k(x,t)F\big(u(t)\big)\,dt,$$

This is a general form of integro-differential equation where $u^{(n)}(x)\ is$ n^{th} derivative of u(x). The kernel k(x,t) and function f(x) are given real value functions and F(u(x)) is a function of u(x). If the limits of integration in above equation are constant then this equation is called Fredholm integro-differential equation and if the atleast one limit in above integral equation is variable then this equation is called Volterra integro-differential equation. Also note that linearity and nonlinearity of above equation is depending on the function F(u(x))[7].

The theory and application of Integro-differential equations play an important role in the mathematical modeling of many fields: physical, biological phenomena and engineering sciences in which it is necessary to take into account the effect of real world problems.

2. Review of Literature and Development in the Subject:

There are many methods in the literature that exists for finding the solutions of integro-differential equations such as the, the variational iteration method (VIM), Homotopy perturbation method (HPM), and Adomian decomposition method (ADM) etc.

Aysegul Akyuz-Dascıoglu and Mehmet Sezer in (2005) and Jafar Biazar and Hamideh Ebrahimi in (2011) solve the system of integro-differential equations by using Chebyshev collocation method and Chebyshev wavelets method resp. [8,9]. Kamel al-Khaled and Fathi Allan (2005) used Decomposition method outlines reliable strategy for solving nonlinear Volterra-Fredhlom integro-differential equation [10]. M. Lakestani and coauthors (2005) developed a technique which is based upon compactly supported linear semiorthogonal B-splin wavelete for solving nonlinear second-order Fredholm integro-differential equations [11]. By using differential transformation P.Darania and Ali Ebadian (2007) solved the linear first order ordinary Fredholm integro-differential equation [12]. Subhra Bhattacharya and B.N. Mandal (2008) presents a method based on polynomial approximation using Bernstein polynomial basis to obtain approximate numerical solution of a singular integro-differential equation with Cauchy kernel [13]. Syed Tauseef Mohyud-Din and N.D. Muhammad Aslam Noor (2009) used He's polynomials for solving higher order integro differential equations (IDES) by converting them to an equivalent system of integral equations [14]. Emran Tohidi,1 M. M. Ezadkhah, and S. Shateyi (2014) used a computational approach for solving a class of nonlinear Volterra integro-differential equations of fractional order which is based on the Bernoulli polynomials approximation [15]. Saheed Alao and coauthors (2014) shows that both ADM and VIM methods are powerful and efficient methods that gives approximations of higher accuracy and closed form solution [16]. Chirag R Trivedi (2015) developed an algorithm of Taylor Collocation Method based on Taylor series for solving various integro differential equations [17]. Olumuyiwa A. Agbolade, and Timothy A. Anake (2017) find numerical solution of Volterra type integro-differential equation of first order with degenerate kernels is obtained by power series collocation method based on two collocating points methods namely Standard Collocation Method (SCM) and Chebyshev-Gauss-Lobatto Collocation Method (CGLCM) [18]. Z. Pashazadeh Atabakan and coauthors (2017) obtained solution of high-order nonlinear Fredholm integro-differential problems by Spectral homotopy analysis method (SHAM)

as a modification of homotopy analysis method (HAM) [19]. Samsun Nahar Ananna and Abdulla – Al – Mamun (2020) introduced some basic idea of Variational iteration method to solve the Volterra's integro-differential equations and investigate the absolute difference between variational iteration method and homotopy perturbation method and draw the graph of difference function by using Mathematica [20].

There are number of mathematicians find the analytical and numerical solution of integro-differential equations by various method. But yet no one work on integro-differential equations involving mixed partial derivatives of first order, second order and higher order.

3. Objectives of Research:

The present study will be a basic, new and original contribution to the theory of integro differential equations. In the present study we will try to find the analytical and numerical solution of some nonlinear partial integro differential equations by using the some improved methods. We will try to solve the some integro-differential equations involving mixed partial derivatives with the help of Laplace substitution method. Also try to solve higher order integro-differential equations and system of integro differential equations involving mixed partial derivatives.

4. Methodology to be Adopted:

There are several methods for study of linear and nonlinear integro differential equations such as HE' Homotopy perturbation method, Parameter expanding method, Laplace substitution method and Runge -Kutta method. We will adopt Laplace substitution method, Modified Laplace substitution method, Variational iterational method and Adomian decomposition method for study the integro-differential equations.

5. Importance of Study:

We hope that the present work will be useful for scientists, engineers, researchers at the international, national level, and mathematicians in their endeavors to analyze the nonlinear problems of the universe in a better way. It has a nice scope for further development in the future. In the future researchers and scientists will be used that theory in different fields of applications. It is claimed that the result of our research work will be a new, basic and original contribution to the theory of nonlinear equations.

6. Proposed Work Plan:

The Year-wise plan of work and targets to be achieved:

Sr. No	Period	Plan of the work	Targets to be achieved
1	First Year	Study of the background required for the research topic, completion of course work.	Good command on the subject and clear the Course work.
2	Second Year	The collection of materials required for this research and work on some problems to be included in the thesis.	Publish two research papers.
3	Third Year	Work on some problems to be included in the thesis.	Publish two research papers.
4	Fourth Year	The research work will be published in journals. The printing and binding of project work. Submission of the thesis will be done this year.	Publishing more research papers and submission of thesis.

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